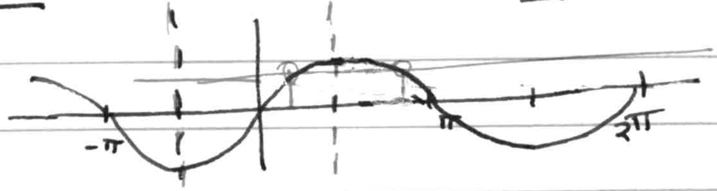
1060 - Day 23
Fall 2013

4.8 - Inverse Trig Functions

<u>Remember</u>: only 1-1 functions have inverses

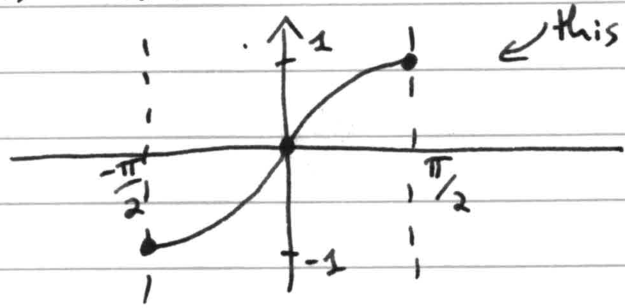
<u>Notice</u>: $f(x) = \sin(x)$ is <u>NOT</u> 1-1



It fails the horizontal line test

<u>Solution</u>: "cut" at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$,
restricting $\sin(x)$ to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

This Gives

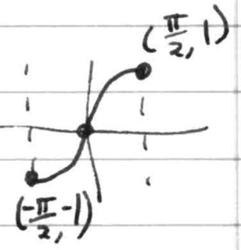


this <u>is</u> one-to-one

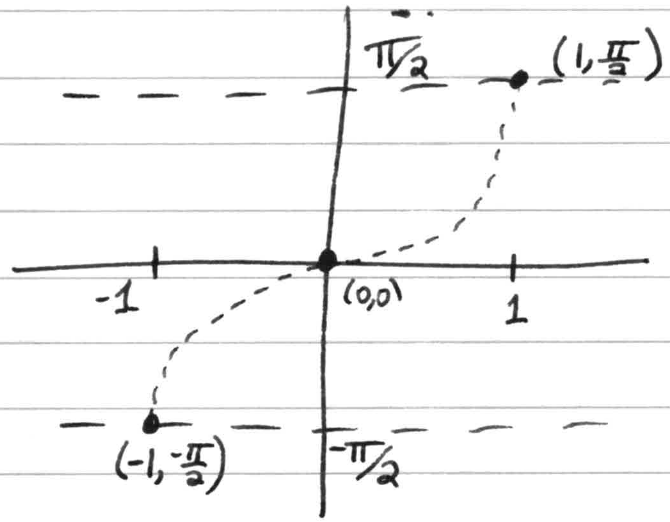
we can get $\sin^{-1}(x) = \arcsin(x)$

Graphically,

by reflecting across $y = x$



(first reflect cut lines)



so: $\sin^{-1}(1) = \arcsin(1) = \frac{\pi}{2}$

$\sin^{-1}(0) = \arcsin(0) = 0$

$\sin^{-1}(-1) = \arcsin(-1) = \frac{-\pi}{2}$

$\sin^{-1}(5)$ DNE.

To compute things like

$$\sin^{-1}\left(\frac{-1}{2}\right)$$

we need a more precise definition of $\arcsin(x) = \sin^{-1}(x)$

Define: $\arcsin(x) = \sin^{-1}(x) = y$

$$\iff$$

$$x = \sin(y)$$

AND

$$y \text{ is in } \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

Eg: compute $\sin^{-1}\left(\frac{-1}{2}\right)$

in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$$\sin(?) = \frac{-1}{2}$$

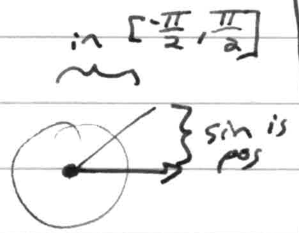
only $\frac{-\pi}{6}$ is IN $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ AND has $\sin\left(\frac{-\pi}{6}\right) = \frac{1}{2}$

sin is neg

So $\frac{-\pi}{6} = \sin^{-1}\left(\frac{-1}{2}\right)$

compute

Eg: $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$



want $\quad \sin(?) = \frac{\sqrt{2}}{2}$

where ? is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

only $\frac{\pi}{4}$ meets both requirements

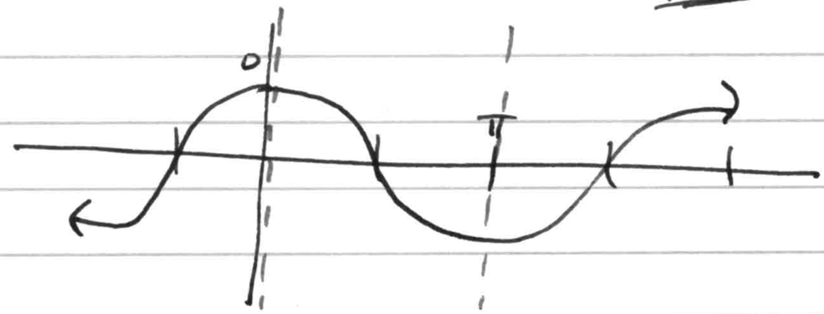
$$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

so

$$\frac{\pi}{4} = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right).$$

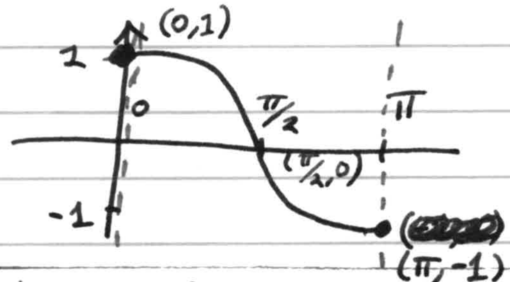
Inverting Cosine

Notice: $f(x) = \cos(x)$ is **NOT** 1-1

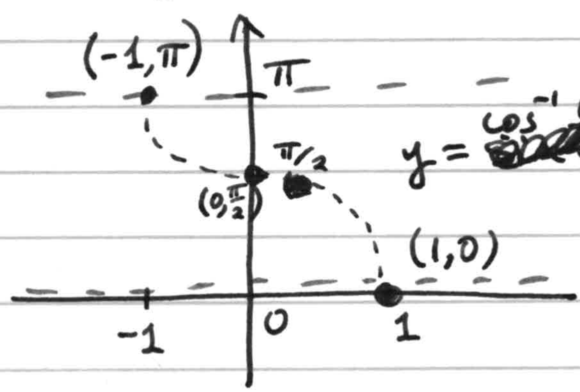


Solution: cut at 0 and π

$\cos(x)$ restricted to $[0, \pi]$ **is** 1-1



To graph the inverse, reflect across $y = x$
(begin w/ cut lines)



$y = \cos^{-1}(x) = \arccos(x)$

Reading off the graph:

$$\cos^{-1}(1) = 0$$

$$\cos^{-1}(0) = \frac{\pi}{2}$$

$$\cos^{-1}(-1) = \pi$$

&

$\cos^{-1}(-2)$ is undefined

---

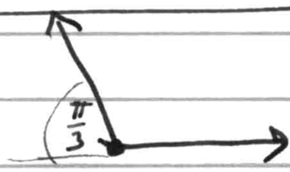
more formally,

$$\boxed{\begin{aligned} \cos^{-1}(x) = \arccos(x) &= y \\ &\Leftrightarrow \\ x &= \cos(y) \\ \underline{\underline{\text{AND}}}\ & y \text{ is in } [0, \pi] \end{aligned}}$$

compute

Eg: $\cos^{-1}\left(-\frac{1}{2}\right)$

$\cos(?) = -\frac{1}{2}$ } in $[0,\pi)$



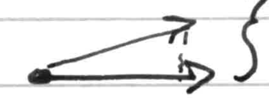
$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$ $\underbrace{\qquad}_{\text{cos is neg}}$

& $\frac{2\pi}{3}$ is in $[0,\pi]$

so $\cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

Eg: Compute
$\arccos\left(\frac{\sqrt{3}}{2}\right)$

$$\boxed{\begin{array}{l} \cos(\,?\,) = \frac{\sqrt{3}}{2} \\ \\ \frac{\pi}{6} \text{ is in } [0, \pi] \\ \quad \underline{\text{AND}} \\ \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \end{array}}$$



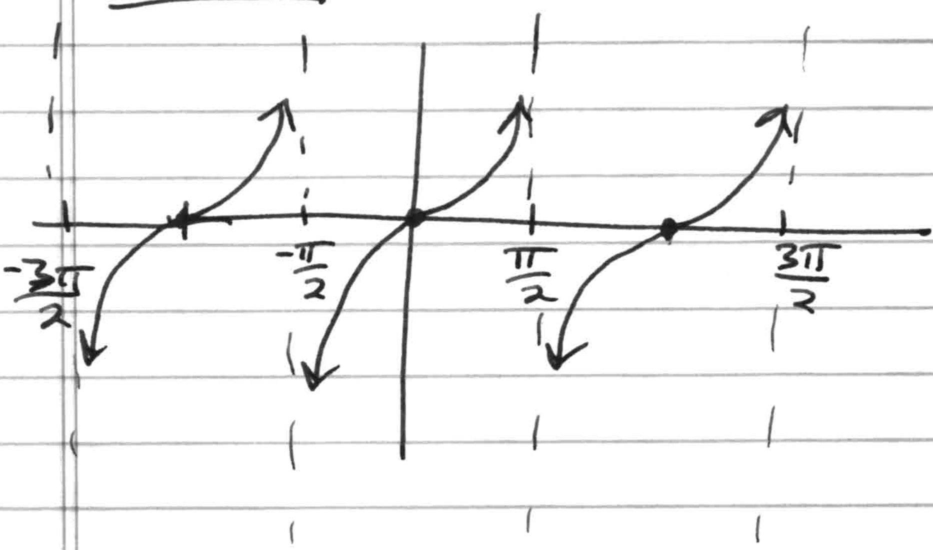
} in $[0, \pi]$

cos is positive

$$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

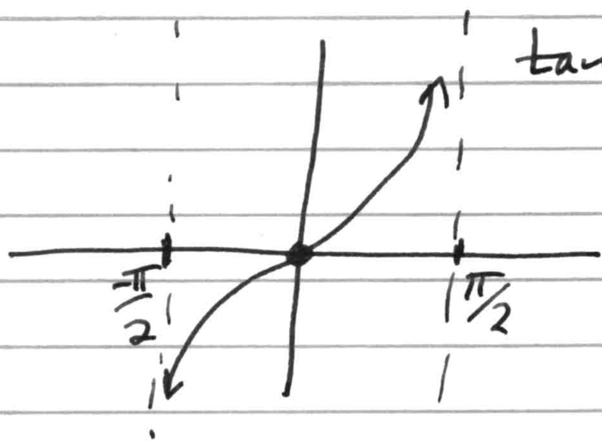
Inverting $f(x) = \tan(x)$

Notice: $f(x) = \tan(x)$ is **NOT** 1-1



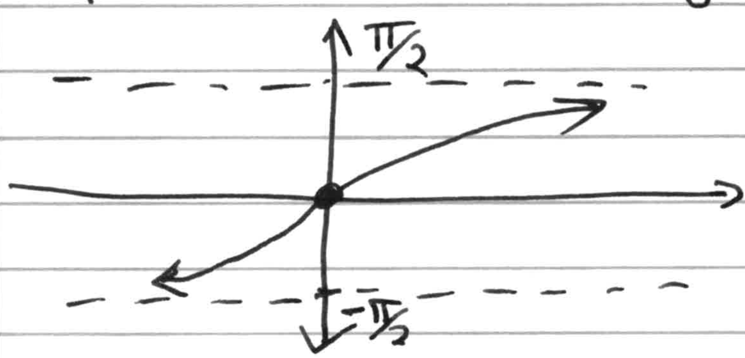
fails horizontal line test

solution "cut" at $-\frac{\pi}{2}$ & $\frac{\pi}{2}$



$\tan(x)$ restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ **is** 1-1

Graph $\tan^{-1}(x) = \arctan(x)$ by reflecting across $y = x$
(reflect cuts first)



Reading off the graph

$\tan^{-1}(0) = 0$

as x gets big & positive
$\tan(x)$ goes to $\frac{\pi}{2}$

as x gets big & negative
$\tan(x)$ goes to $\frac{-\pi}{2}$

<u>Precisely</u>:

$$\boxed{\begin{array}{c} \tan^{-1}(x) = y \\ \Longleftrightarrow \\ x = \tan(y) \\ \text{AND } y \text{ is in } \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \end{array}}$$

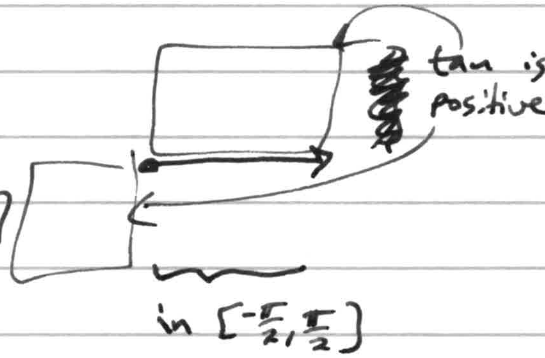
<u>Eg</u>: compute $\tan^{-1}(1)$

$\tan(?) = 1$

$\frac{\pi}{4}$ is in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

and

$\tan\left(\frac{\pi}{4}\right) = 1$



$\tan^{-1}(1) = \frac{\pi}{4}$

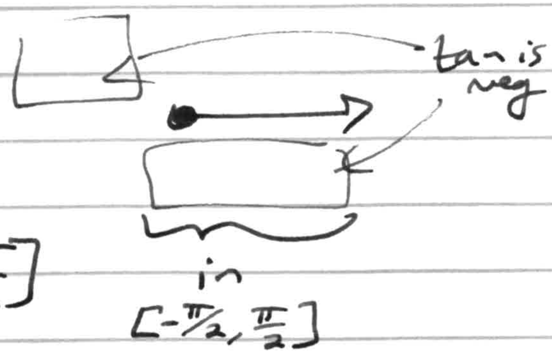
Eg: compute $\tan^{-1}(-1)$

$$\tan(\,?\,) = -1$$

$$\tan\left(\frac{-\pi}{4}\right) = -1$$

and

$\frac{-\pi}{4}$ is in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$



$$\tan^{-1}(-1) = \frac{-\pi}{4}$$

See textbook
for definition & graph
of $\sec^{-1}(x)$
